

## Quick Notes on Solving Quadratics by Completing the Square

Goal: Be able to solve quadratic equations like  $2x^2+10x+6=0$  by *completing the square*.

Vocab: Completing the Square, Squared Binomial, Perfect Square Trinomial

First, you need to know the difference between a *Squared Binomial* and a *Perfect Square Trinomial*. The two expressions at the right are equivalent, the second was found by multiplying (FOIL) the first.

$$\text{Squared Binomial: } (x+7)^2$$

$$\text{Perfect Square Trinomial: } x^2+14x+49$$

So you can just multiply a squared binomial to convert it into a perfect square trinomial (PST).

$$(x-8)^2=(x-8)(x-8)=x^2-16x+64$$

Before you can use 'Completing the Square,' you have to be able to go back the other way:

$$x^2+18x+81\Rightarrow(x+9)^2$$

Look at the four examples on the right. Notice any patterns?

$$x^2+12x+36=(x+6)^2$$

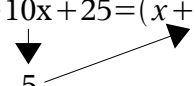
$$x^2-14x+49=(x-7)^2$$

$$x^2+20x+100=(x+10)^2$$

$$x^2-26x+169=(x-13)^2$$

Three things to notice: 1) The number front of the  $x^2$  is always 1. 2) If you divide the number in front of the  $x$  by two, then square it, you always get the constant. 3) Half of the number in front of  $x$  is always the number in the parentheses.

So you can use those facts to check to see if a number is a PST, then re-write it as a squared binomial.

$$x^2+10x+25=(x+5)^2$$


Now you're ready to solve quadratic equations using completing the square.

$$\text{Solve: } x^2+24x+30=0$$
$$\qquad\qquad +114 \quad +114$$

$$x^2+24x+144=114$$

$$(x+12)^2=114$$

$$x+12=\pm\sqrt{114}$$

$$x=-12\pm\sqrt{114} \quad \text{Done.}$$

The expression on the left is **not** a PST, so determine what must be added to it so it is a PST. Half of 24 is 12, then squared, is 144. We'll need to add 114 to both sides.

Now you can re-write the left side as a squared binomial.

Square root both sides. Don't forget both the positive and negative roots of 114.\*

Now get  $x$  by itself.

Here's another: Solve  $x^2 - 12x - 10 = 0$

You need 36 to the right of the  $-12x$ . I'll add 46 to each side.

$$\begin{aligned}x^2 - 12x + 36 &= 46 \\(x - 6)^2 &= 46 \\x - 6 &= \pm\sqrt{46} \\x &= 6 \pm\sqrt{46}\end{aligned}$$

What if there's a number in front of  $x^2$ ?

Solve:  $2x^2 + 28x - 13 = 0$

$$x^2 + 14x - \frac{13}{2} = 0$$

$$x^2 + 14x = \frac{13}{2}$$

$$x^2 + 14x + 49 = \frac{13}{2} + 49 = \frac{13}{2} + \frac{98}{2} = \frac{111}{2}$$

$$x^2 + 14x + 49 = \frac{111}{2}$$

$$(x + 7)^2 = \frac{111}{2}$$

$$x + 7 = \pm\sqrt{\frac{111}{2}}$$

$$x = -7 \pm\sqrt{\frac{111}{2}}$$

Some people like to get the  $-10$  out of the way to the other side,  $x^2 - 12x = 10$ , then figure out what to add to each side. Do it either way is comfortable.

Drop the left down to a squared binomial, square root both sides (don't forget plus and minus), then isolate  $x$ .

First, divide the equation by the number in front.

Next, get the constant out of the way. (You can do these steps in either order.)

Add 49 to both sides (half of 14, squared), then simplify.

Now continue as before, being careful with the fractions.

Yes, the  $\sqrt{\frac{111}{2}}$  can be simplified to  $\frac{\sqrt{222}}{2}$  if you wish, and know how. Then you could combine it with the  $-7$  as  $-\frac{14}{2}$  to get  $\frac{-14 \pm \sqrt{222}}{2}$ . Fine with me.

\*Note to those who teach math: I realize that going from  $(x + 12)^2 = 114$  to  $x + 12 = \pm\sqrt{114}$  is not technically correct. To write it correctly, it should be

$$\begin{aligned}(x + 12)^2 &= 114 \\|x + 12| &= \sqrt{114} \\x + 12 &= \pm\sqrt{114}\end{aligned}$$

I find it helps students understanding of the overall picture to have them simply remember that there are two possibilities for the roots of the number. Then, after we've spent more talking about roots in general, I go back and point out that  $\sqrt{x^2} = |x|$ .

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