

Properties of Exponents

Property Name	Property Statement	Examples
Zero Exponent	$b^0 = 1$	$2^0 = 1, x^0 = 1$
Negative Exponent	$b^{-n} = \frac{1}{b^n}$	$X^{-3} = \frac{1}{X^3}, a^{-4}b = \frac{b}{a^4}, \frac{1}{k^{-3}} = \frac{1}{\frac{1}{k^3}} = k^3, \frac{x^3y^{-2}}{z^{-4}} = \frac{x^3z^4}{y^2}$
Product of Powers	$b^m \cdot b^n = b^{m+n}$	$g^3 \cdot g^5 = g^8, a \cdot a^3 \cdot a^4 = a^8, c^3r^4 \cdot c^2r^6t = c^5r^8t$
Quotient of Powers	$\frac{b^m}{b^n} = b^{m-n}$	$\frac{3^6}{3^2} = 3^4, \frac{x^2y^4}{y^2} = x^2y^2, \frac{w^3x^2}{wx^5} = w^2x^{-3}$
Power of a Power	$(b^m)^n = b^{mn}$	$(a^3)^4 = a^{12}, (p^{-2})^3 = p^{-6}$
Power of a Product	$(ab)^n = a^n b^n$	$(kt)^4 = k^4t^4, (a^2g^4)^3 = a^6g^{12}, (4v^2y^3)^2 = 16v^4y^6$
Power of a Quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{h}{k}\right)^4 = \frac{h^4}{k^4}, \left(\frac{3x^3}{y^4z^2}\right)^5 = \frac{243x^{15}}{y^{20}z^{10}}$

The properties shown above are the main ones shown in any textbook. The following, although not a property, is something helpful that can be shown from the other properties:

$\left(\frac{a}{b}\right)^{-n} = \frac{a^{-n}}{b^{-n}} = \frac{b^n}{a^n} = \left(\frac{b}{a}\right)^n$. In other words, if a fraction has a negative exponent, then you can simply flip over the fraction and change the negative exponent to a positive exponent of the same value.

Further examples:

(Note: expressions with negative exponents are not considered simplified.)

$$\text{Ex. 1: } \left(\frac{3a^2b^3}{a^5b}\right)^3 = \left(\frac{3b^2}{a^3}\right)^3 = \frac{3^3b^6}{a^9} = \frac{27b^6}{a^9}$$

$$\text{Ex. 2: } (x^{-3}y^4z^{-1})^3 = \left(\frac{y^4}{x^3z}\right)^3 = \frac{y^{12}}{x^9z^3}$$

$$\text{Ex. 3: } \left(\frac{3m^{-2}t^2}{12}\right)^3 = \left(\frac{m^{-2}t^2}{4}\right)^3 = \left(\frac{t^2}{4m^2}\right)^3 = \frac{t^6}{4^3m^6} = \frac{t^6}{64m^6}$$

$$\text{Ex. 4: } \left(\frac{2b^{-2}c^3d^{-2}}{6c^7d^5}\right)^{-2} = \left(\frac{1}{3b^2c^4d^7}\right)^{-2} = \left(\frac{1}{3b^2c^4d^7}\right)^{-2} = \left(\frac{3b^2c^4d^7}{1}\right)^2 = 9b^4c^8d^{14}$$